Lecture 4: Topics in Dynamic Mechanism Design

NMI Workshop, ISI Delhi

August 4, 2015

Lecture Plan

I will cover a few different models.

The informal statements of the main results of the paper will be presented with some intuition.

Issues covered will be efficiency with interdependent values, dynamic insurance and limited commitment.

Deb and Said (forthcoming): Dynamic Screening with Limited Commitment

Journal of Economic Theory

Motivation

Two assumptions are made often in dynamic mechanism design models.

1. Agents cannot choose when to enter the contract.
2. The principal has full commitment.

Relax both assumptions in a sequential screening setting.

- Assume buyers in the first period can delay contracting.
- Seller cannot commit to second period terms.

Model: Cohort-one buyers

There is a unit mass of buyers who enter in period one.

Cohort-one buyers have a period-one type $\lambda \in \Lambda \subseteq \mathbb{R}_+$.

- $\lambda \sim F$, with continuous and positive density f.

In period two, each buyer learns her value $v \in \mathbf{V} \subseteq \mathbb{R}_+$.

- $v \sim G(\cdot|\lambda)$, with density $g(\cdot|\lambda)$.
- $\{G(\cdot|\lambda)\}_{\lambda \in \Lambda}$ is ordered by FOSD.

Model: Cohort-two buyers

There is a mass $\gamma > 0$ of new entrants in period two.

Each cohort-two buyer knows her value $v \in \mathbf{V}$.

- $v \sim H$, with continuous and positive density h.

We use p_H to denote the monopoly price corresponding to these late-arriving buyers:

$$p_H := \underset{p}{\operatorname{argmax}}\{(p - c)(1 - H(p))\}.$$

p_H is also the monopoly price for some $\hat{\mu} \in \Lambda$:

$$p_H = p_{\hat{\mu}} := \underset{p}{\operatorname{argmax}}\{(p - c)(1 - G(p|\hat{\mu}))\}.$$

Contracts

Seller offers a sequential screening contract in period one.

- Seller can commit to the terms of this contract.
- Cohort-one do not have to contract.

Seller offers a price in period two.

- Determined sequentially rationally.
- Is a type dependent outside option in period one.

Limited commitment

- Many possible interpretations of "cannot pre-commit."
- Implications depend on richness of permissible language.
- At least three broad categories:
  - Contracts that using explicit penalties for deviation.
  - Best-price guarantees, most-favored-nation clauses, and other implicit commitment contracts.
  - Contracts where period-one options cannot condition on the period-two mechanism, either directly or indirectly.
- Easy to argue that explicit penalties can achieve the full-commitment optimum.
- Other extreme: limited commitment with no (explicit or implicit) conditioning.

Main tradeoff

- There is a simple strategic tension:
  - The seller would like to sell to cohort-one buyers using dynamic screening contracts.
  - The seller would also like to sell to cohort-two buyers by offering a "last-minute" static contract.
- But cohort-one buyers may be able to avail themselves of this second-period contract...
- ...and the seller cannot prevent these buyers from purchasing in period two.
  - Buyers are often anonymous until a transaction is made.
  - Or regulations/custom may prevent identity-based price discrimination.
  - "Advance purchasers" may even be able to break their contracts and buy at the last minute instead.

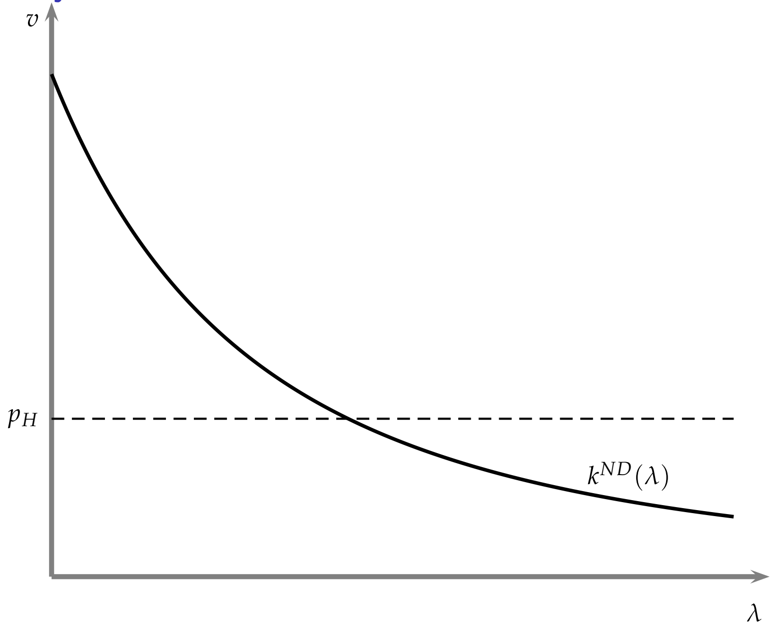
Strategic delay

- The period-two contract affects the seller's ability to screen cohort-one buyers.
  - It creates an endogenous outside option in period one.
  - This participation constraint limits the seller's ability to extract rents.
- If the seller can commit to a high future price, the outside option becomes unattractive and cohort-one profits rise.
- But with limited commitment, the period-one seller is now competing with his future self.

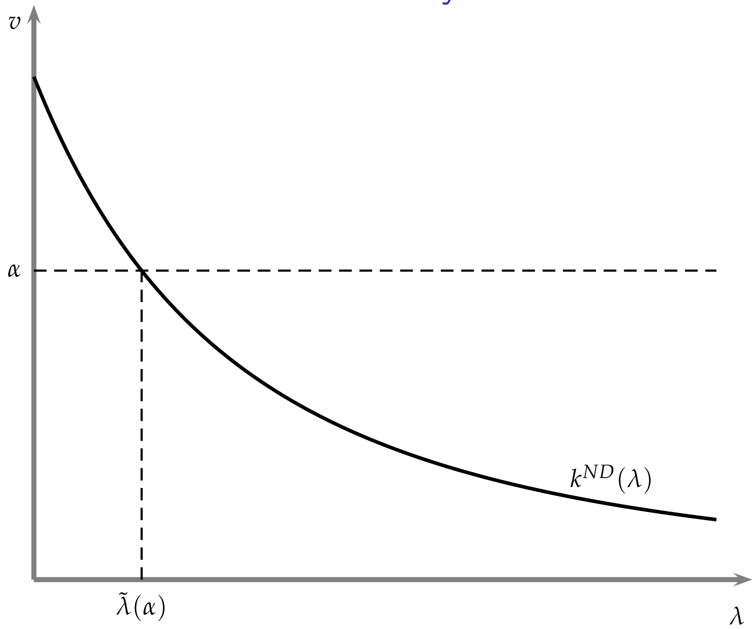
Role of commitment

- Suppose that the mass γ of cohort two is small.
- And suppose the cohort-two monopoly price p_H is low.
  - Then waiting is very attractive for buyers in cohort one.
  - The seller's ability to extract rents in period one is reduced.
- But the cohort-two contribution to total profits is small.
- This creates an incentive to "manage" demand and encourage delay.
  - By delaying contracting, the seller can generate stronger period-two demand.
  - This leads to a higher period-two price (and a lower period-one outside option).

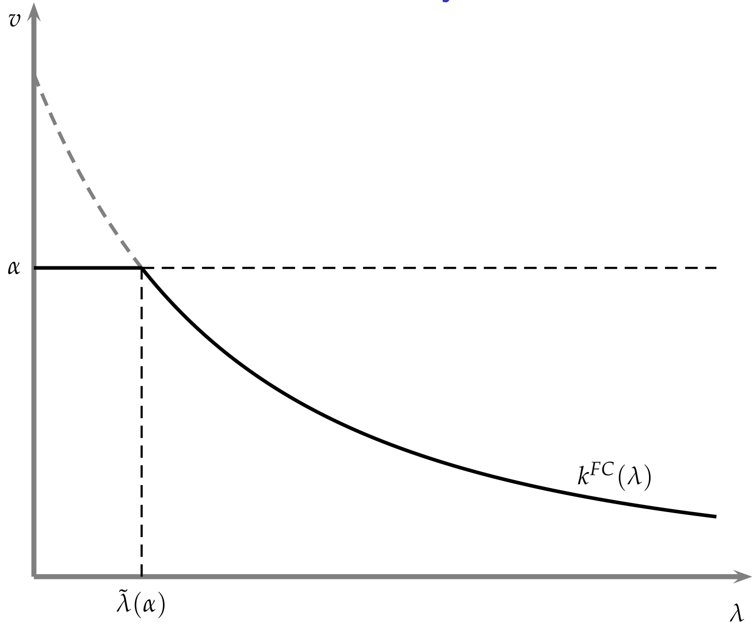
Courty Li Benchmark



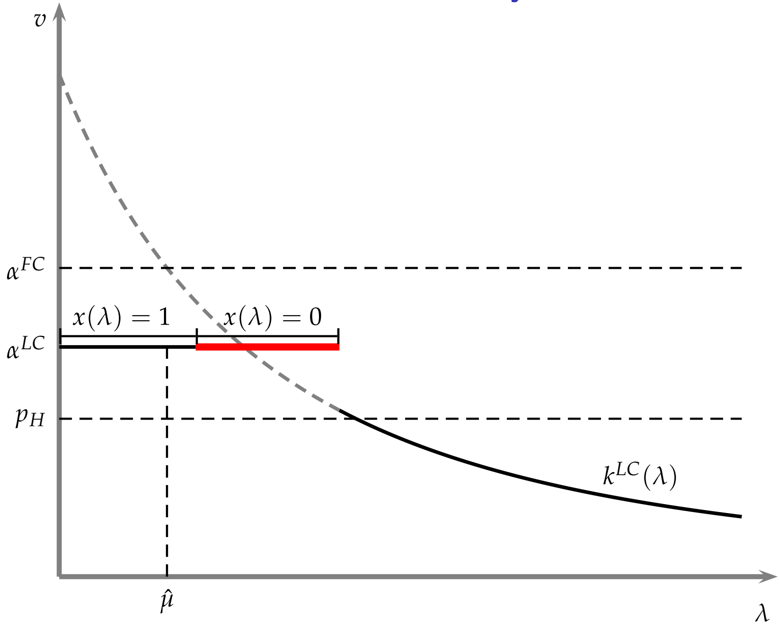
Full Commitment and Delay



Full Commitment and Delay



Limited Commitment and Delay



Bergemann and Välimäki (2010): The Dynamic Pivot Mechanism

Econometrica

Efficient Dynamic Mechanisms

- Generalize the idea of VCG to dynamic environments with private information.
- In these environments, agents receive new private information in each period.
- The distribution of future private values are correlated with the current value and the outcome.
- Can an intertemporal sequence of transfer payments implement the efficient outcome?

Dynamic Pivot Mechanism

- Allows each agent to receive her flow marginal contribution in every period.
  - After each history, the expected transfer coincides with the dynamic externality cost that an agent imposes on the other agents.
- The dynamic pivot mechanism is socially efficient.
- Periodic ex post implies with respect to information in period t.
- It satisfies (periodic) ex post incentive constraints.
- It satisfies (periodic) ex post participation constraints.

Scheduling Example

- Scheduling tasks
- Discrete time, infinite horizon: $t = 0, 1, \ldots$.
- Common discount factor δ.
- Finite number of agents: $i \in \{1, \ldots, I\}$.
- Each agent i has a single task
- Value of task for i is: $v_i > 0$.
- Quasilinear utility: $v_i - p_i$.
- Assume that wlog

$$v_1 > \cdots > v_I.$$

Efficient Assignment

- Efficient assignment: Task 1 is scheduled in Period 0, Task 2 is scheduled in Period 1 and so on.
- Marginal contribution M_i of i from time 0 perspective is:

$$
\begin{aligned}
M_i &= \sum_{t=1}^{I} \delta^{t-1} v_t - \left(\sum_{t=1}^{i-1} \delta^{t-1} v_t + \sum_{t=i}^{I-1} \delta^{t-1} v_{t+1} \right) \\
&= \sum_{t=i}^{I} \delta^{t-1} (v_t - v_{t+1}) \\
&\geq 0
\end{aligned}
$$

Pricing

- At time $t = i - 1$, agent i completes task and realizes value v_i.
- Externality cost of agent i is equal to the next valuable task v_{i+1} minus the improvement in future allocations due to the delay of all tasks by one period.
- Marginal contribution to externality pricing is

$$p_i = v_{i+1} - \sum_{t=i+1}^{I} \delta^{t-i}(v_t - v_{t+1}) = (1 - \delta) \sum_{t=i}^{I} \delta^{t-i} v_{t+1}$$

- The externality cost of agent i in the intertemporal framework is less than in the corresponding single allocation problem where it would be v_{i+1}.

Incomplete Information

- v_i is private information to agent i at $t = 0$.
- IC and efficient sorting: when would i like to win against $j-1$ versus j where $j-1 \geq i$

$$(v_i - v_j) - \sum_{t=j}^{I} \delta^{t-(j-1)}[v_t - v_{t+1}]$$

$$\geq \delta(v_i - v_{j+1}) - \sum_{t=j+1}^{I} \delta^{t-j+1}[v_t - v_{t+1}].$$

- This reduces to

$$(1-\delta)v_i \geq (1-\delta)v_j.$$

General Model

- Agents are denoted by $i = 1, \ldots, I$ and time by $t = 0, 1, \ldots$.
- Agents have a common discount factor $\delta \in (0, 1)$.
- The quasilinear flow utility of agent i in period t is

$$v_i(a_t, \theta_{i,t}) - p_{i,t}.$$

- Allocation $a_t \in A$ where A is finite.
- Markovian state $\theta_t = (\theta_{1,t}, \ldots, \theta_{I,t}) \in \Theta$.
- $p_{i,t}$ is the monetary transfer.
- The private (Markovian) signal $\theta_{i,t+1}$ is generated by a conditional distribution function:

$$\theta_{i,t+1} \sim F_i(\cdot | a_t, \theta_{i,t}).$$

Social Optimum

- The socially optimal programme at period t in state θ_t is

$$W(\theta_t) = \max_{\{a_s\}_{s=t}^{\infty}} \mathbb{E}\left[\sum_{s=t}^{\infty} \delta^{s-t} \sum_{i=1}^{I} v_i(a_s, \theta_{i,s})\right].$$

and the solution to this programme $\mathbf{a}^* = \{a_t^*\}_{t=0}^{\infty}$.

- In recursive form, this is

$$W(\theta_t) = \max_{a_t} \mathbb{E}\left[\sum_{i=1}^{I} v_i(a_t, \theta_{i,t}) + \delta \mathbb{E} W(\theta_{t+1})\right].$$

- Social value in absence of i is

$$W_{-i}(\theta_t) = \max_{\{a_s\}_{s=t}^{\infty}} \mathbb{E}\left[\sum_{s=t}^{\infty} \delta^{s-t} \sum_{j \neq i} v_j(a_s, \theta_{j,s})\right].$$

and the solution to this programme $\mathbf{a}_{-i}^* = \{a_{-i,t}^*\}_{t=0}^{\infty}$.

Histories

- We consider direct mechanisms which truthfully implement the socially efficient policy.
  - The principal does not require commitment.
- Agent i reports $r_{i,t} \in \Theta_i$ in period t.
- In this setting, public histories are

$$h_t = (r_0, a_0, r_1, a_1, \ldots, r_{t-1}, a_{t-1})$$

  where r_s is the vector of reports.
- It is assumed that past reports are observed by all.
- Private histories are therefore

$$h_{i,t} = (\theta_{i,0}, r_0, a_0, \theta_{i,1}, r_1, a_1, \ldots, \theta_{i,t-1}, r_{t-1}, a_{t-1}, \theta_{i,t}).$$

Dynamic Efficient Mechanisms and Strategies

- An efficient dynamic direct mechanism is a family of allocations and transfers

$$a_t^* : \Theta \to \Delta(A) \text{ and } p_t : H_t \times \Theta \to \mathbb{R}^I.$$

- Socially efficient allocations do not depend on previous reports but transfers may.
- Strategy for i in t is a mapping from the private history to the report space:

$$r_{i,t} : H_{i,t} \to \Theta$$

and the complete strategy is denoted by $\mathbf{r}_i = \{r_{i,t}\}_{t=0}^{\infty}$.

IC+IR

- Given the mechanism $\{a_t^*, p_t\}_{t=0}^{\infty}$ and reporting strategies $\mathbf{r}_{-i}$, the optimal strategy for i is

$$V_i(h_{i,t}) = \max_{r_{i,t} \in \Theta_i} \mathbb{E}\left\{v_i(a_t^*(r_{i,t}, r_{-i,t}), \theta_{i,t}) - p_i(h_t, r_{i,t}, r_{-i,t}) + \delta V_i(h_{i,t+1})\right\}.$$

- $V_i(h_{i,t})$ is the continuation value of i at $h_{i,t}$.
- A dynamic direct mechanism is interim IC if for every agent and every history, truthtelling is a best response to truthtelling by others.
- A dynamic direct mechanism is periodic ex post IC if truthtelling is a best response regardless of history & current state of the others.
- Periodic as reports aren't ex post IC with respect to signals arriving after period t.
  - i may receive information at some $s > t$ which may make her want to change report at t.
- Periodic IR implies that at each history h_t, i can leave the mechanism.

Marginal Contribution

- The marginal contribution $M_i(\theta_t)$ of agent i at θ_t is

$$M_i(\theta_t) = W(\theta_t) - W_{-i}(\theta_t).$$

- The flow marginal contribution $m_i(\theta_t)$ is

$$\begin{aligned} M_i(\theta_t) &= m_i(\theta_t) + \delta \mathbb{E} M_i(\theta_{t+1}) \\ \Longrightarrow m_i(\theta_t) &= W(\theta_t) - W_{-i}(\theta_t) - \delta \mathbb{E}[W(\theta_{t+1}) - W_{-i}(\theta_{t+1})]. \end{aligned}$$

- A monetary transfer which makes the flow net utility match the flow marginal contribution makes agent i internalize her social externalities:

$$p_i^*(\theta_t) = v_i(a_t^*, \theta_{i,t}) - m_i(\theta_t).$$

Dynamic Pivot Mechanism

Theorem

The dynamic pivot mechanism is periodic ex post incentive compatible and periodic individually rational.

Heng Liu (2015): Efficient Dynamic Mechanisms in environments with Interdependent Valuations

Working Paper

Overview

Studies efficient allocation in dynamic environments with interdependent types and changing private information.

Extends the results of Bergemann and Välimäki (2011) to allow for correlation.

Essentially, leverages the dynamic equivalent of the Crémer and Mclean (1988) result.

Example: Repeated Auction

Two firms, compete for two licenses to drill for oil on two adjacent areas.

The licenses are sold sequentially via two auctions ($t \in \{1, 2\}$).

Allocation: $a_t \in \{1, 2\}$.

Payoff from obtaining license:

$$v_1(s_t) = 2s_t - 1,\ v_2(s_t) = 3s_t - 6,$$

where s_t is the oil quantity.

Firms maximize sum of profits and there is no discounting.

Example: Information

Prior to auction t, firm t gets a signal $\theta_t = s_t \in \{4, 6\}$.

- Firms get signals in different periods.

Joint Distribution

$$f(\theta_1, \theta_2) = \begin{bmatrix} f(4,4) & f(4,6) \\ f(6,4) & f(6,6) \end{bmatrix} = \begin{bmatrix} 3/8 & 1/8 \\ 1/8 & 3/8 \end{bmatrix}$$

Conditional Distribution

$$f(\theta_2 \,|\, \theta_1) = \begin{bmatrix} f(4|4) & f(4|6) \\ f(6|4) & f(6|6) \end{bmatrix} = \begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix}$$

Example: First Auction

$$v_1(s_t) = 2s_t - 1,\ v_2(s_t) = 3s_t - 6,\ f(\theta_2 \mid \theta_1) = \begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix}$$

Efficient Outcome: $a_1^* = \begin{cases} 1 & \text{if } \theta_1 = 4, \\ 2 & \text{if } \theta_1 = 6. \end{cases}$

Not IC: $\begin{array}{l} 2 \times 4 - 1 - p_1^1(4) \geq 0 - p_1^1(6) \\ 0 - p_1^1(6) \geq 2 \times 6 - 1 - p_1^1(4) \end{array} \implies 4 \geq 6.$

Example: Second Auction

$$v_1(s_t) = 2s_t - 1,\ v_2(s_t) = 3s_t - 6,\ f(\theta_1 \,|\, \theta_2) = \begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix}$$

Efficient Outcome: $a_2^* = \begin{cases} 1 & \text{if } \theta_2 = 4, \\ 2 & \text{if } \theta_2 = 6. \end{cases}$

Consider Payment: $p_2^2 = \begin{cases} 0 & \text{if } r_2 = 4, \\ 11 & \text{if } r_2 = 6. \end{cases}$

Can be implemented statically: $\begin{array}{c} 0 - 0 \geq 3 \times 4 - 6 - 11 \\ 3 \times 6 - 6 - 11 \geq 0 - 0 \end{array}.$

Example: Linking Payments

$$v_1(s_t) = 2s_t - 1,\ v_2(s_t) = 3s_t - 6,\ f(\theta_2 \mid \theta_1) = \begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix}$$

Firm 1 is only asked to make a payment at 2:

$$p_2^1 = \begin{cases} -4 & \text{if } a_1 = 2, r_2 = 4 \\ -16 & \text{if } a_1 = 2, r_2 = 6 \\ 0 & \text{otherwise} \end{cases}$$

IC is satisfied: $$\begin{array}{l} 2 \times 4 - 1 - 0 \geq 0 + \left(\frac{3}{4} \times 4 + \frac{1}{4} \times 16\right) \\ 0 + \left(\frac{1}{4} \times 4 + \frac{3}{4} \times 16\right) \geq 2 \times 4 - 1 - 0 \end{array}.$$

Payment for 2 as on the previous slide.

Model

The buyer's utility now depends on all the types $v_i(a_t, \theta_t) - p_i$.

The Markov transitions are given by $\theta_{t+1} \sim F_i(\cdot \mid a_t, \theta_t)$.

Mechanisms are still:

$$a_t : \Theta_t \to \Delta(A) \text{ and } p_t : H_t \times \Theta_t \to \mathbb{R}^I.$$

Results: Finite Horizon

Periodic Ex-Post IC: truth-telling is a beast response regardless of private history and the current signals of other agents, given that other agents adopt truthful strategies.

Assume intertemporal correlation condition (a la Crémer and McLean 1988).

Consider first a finite horizon T.

Assume the efficient allocation is implementable in period T.

There exists a periodic ex-post IC efficient dynamic mechanism (where payments only depend on today and yesterday's report, allocation).

Results: Infinite Horizon

Again assume intertemporal correlation condition.

Now consider $T = \infty$.

There exists a periodic ex-post IC efficient dynamic mechanism that balances the budget in the truthful equilibrium.

Farinha Luz (2015): Dynamic Competitive Insurance
Working Paper

Overview

Studies the dynamic coverage and premiums in long-term relationships between insurance buyers and providers.

The model considered is dynamic extension of Rothschild and Stiglitz (1976).

- Allows for perfectly competitive insurance market.

In contrast to the literature, the model allows for risk aversion.

The dynamics of coverage are similar to the setting of Battaglini (2005).

The Model

A single agent lives for $T \in \{1, \ldots, \infty\}$ periods.

At the beginning of each period t, the agent (privately) observes her type $\theta_t \in \Theta := \{\theta_l, \theta_h\}$.

The type determines a probability distribution over realized income $y_t \in Y$.

- y_t is publicly observed at the end of t and contractible.
- $y \sim p_\theta$.
- $\sum_{y \in Y} p_{\theta_l} y < \sum_{y \in Y} p_{\theta_h} y$: θ_h is the good type.

Types follow a Markov chain.

The Model Continued

Period t consumption preferences are determined by strictly concave and strictly increasing $u : \Re_+ \to \Re$.

- This captures risk aversion.

The utility obtained from deterministic consumption stream $(c_1, \ldots, c_T)$ is $\sum_{t=1}^{T} \delta^{t-1} u(c_t)$.

There is an initial-type dependent outside option $(\underline{V}_h, \underline{V}_l)$.

The payoff to a firm is given by $\sum_{t=1}^{T} \delta^{t-1}(y_t - c_t)$.

Contracts

Single period insurance contracts: $C := \{c : Y \to \Re_+\}$.

Dynamic contracts are given by the mapping $c_t : H_r^t \to C$, where $H_r^t := \{r_1, \ldots, r_t\}$ is the history of type reports.

c_t is realization independent as it does not depends on history on income realizations.

Timing:

1. All $N \geq 2$ firms simultaneously offer long-term contracts.
2. The agent observes θ_1 and chooses a contract.
3. If the buyer does not accept any offer, he receives payoff $\underline{V}_{\theta_1}$.

Two sided full commitment.

Complete Information Benchmark

Risk types are observed by all firms.

Firms compete for both types and all profits are eliminated.

Each type is offered full insurance up front: $\mathbb{E}\left[\sum_{t\geq 1} \delta^t y_t \mid \theta_1\right]$.

Full observability of future risk types is not required.

- The same outcome arises if firms can only observe θ_1.

Profit Maximizing Contracts

Find contracts that give the buyer an incentive compatible ex-ante payoff at minimal cost.

The profit maximizing contract involves complete coverage except at histories that only involve consecutive high-type announcements.

The continuation contract for $\theta_1 = \theta_h$ displays distortions (partial insurance) to prevent type θ_l from misreporting.

- The incentives are provided in the future.
- Consecutive announcements of θ_h serve as a signal of an initial period type θ_h.
- Compare with Battaglini (2005).

Competition drives profits to 0.

Thanks to the organizers!